

Deriving the Quadratic Formula  
*from*  
Completing the Square

<https://www.chilimath.com/lessons/intermediate-algebra/derive-quadratic-formula/>

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Step 1:** Let  $y = 0$  in the general form of the quadratic function  $y = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers but  $a \neq 0$ .

$$ax^2 + bx + c = 0$$

- **Step 2:** Move the constant  $c$  to the right side of the equation by subtracting both sides by  $c$ .

$$\begin{aligned} ax^2 + bx + c - c &= 0 - c \\ ax^2 + bx &= -c \end{aligned}$$

- **Step 3:** Divide the entire equation by the coefficient of the squared term which is  $a$ .

$$\frac{1}{a}(ax^2 + bx = -c)$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

- **Step 4:** Now identify the coefficient of the linear term  $x$ .

**Coefficient of the linear term**



$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

- **Step 5:** Divide it by 2 and raise it to the 2nd power. Then simplify it further.

**Square it**

$$\left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

**Divide by 2**

- **Step 6:** Add the output of step #5 to both sides of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

- **Step 7:** Simplify the right side of the equation.

Simplify the **right side** of the equation



$$\begin{aligned}x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{-c}{a} \left( \frac{4a}{4a} \right) + \frac{b^2}{4a^2} \\ &= \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} \\ x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{b^2 - 4ac}{4a^2}\end{aligned}$$

- **Step 8:** Express the trinomial on the left side of the equation as the square of a binomial.

$$\left( x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2}$$

- **Step 9:** Take the square root of both sides of the equation to eliminate the exponent 2 of the binomial.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

- **Step 10:** Simplify. Make sure that you attach the  $\pm$  on the right side of the equation. The left side no longer contains the power 2.

**Don't forget to attach the plus or minus symbol on the right side of the equation.**

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

- **Step 11:** Keep the variable  $x$  on the left side by subtracting both sides by  $\frac{b}{2a}$ .

$$x + \frac{b}{2a} - \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

- **Step 12:** Simplify and we are done!

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$